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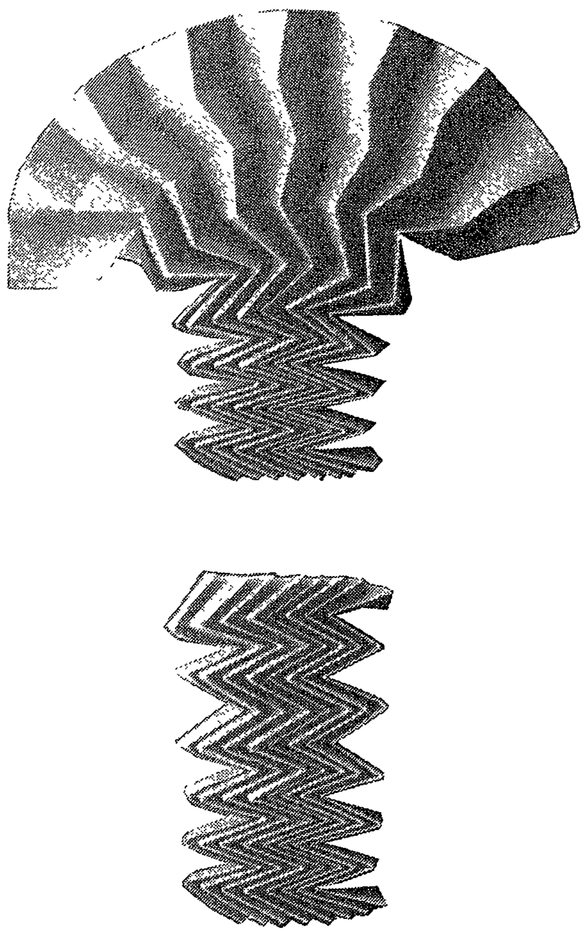

The *Miura-ori*
opened out like a fan

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EVOLUTION OF ORIGAMI ORGANISMS

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Abstract: *This article (based my book* Viva! Origami *(Maekawa, 1983) will show origami design as the tiling work. First, the rules of origami and some of the notions of origami taxonomy will be considered. Furthermore, traditional models will be analyzed; the meaning of 'basic form' will be discussed; and an introduction to my original designs will be explained.*

1 ORIGIN OF THE SYSTEMATIC STRUCTURE

In origami, an organism is regarded as a transformed flexible sheet. This sheet is able to be split and fused keeping the distinction between its surface and its reverse side (Kasahara and Takahama). Origami folding begins with a sheet of paper, which is transformed by folds. Of course, there are exceptions in that sometimes several sheets of paper are used, and, on occasions, scissors. However, for the moment, we shall consider the strict and traditional rules of origami. Most of paper-folders have implicit rules. The following are "the five commandments" by Husimi, arranged by Kasahara (1989). These are typical rules of origami.

1. Start with a sheet of square paper.
2. Cutting and gluing are forbidden.
3. Fold model flat just before its completion.
4. Straight folding is only permitted.
5. When constructing a model, bear in mind the physical qualities of paper.

I think Husimi made his rules from the character of traditional origami. Most classical origami models (in documents) violate all of the above rules. Models that adhere to the rules have been handed down by tradition, and are not described in any extant document. These models are the result of historical selection over a 1000 years. (I describe 'over a 1000 years', but there is no established view when origami had its beginnings. This is my guess by the introduction of paper.) It can be compared to natural selection. In this analogy, the most important question to be considered is: What is selection pressure?

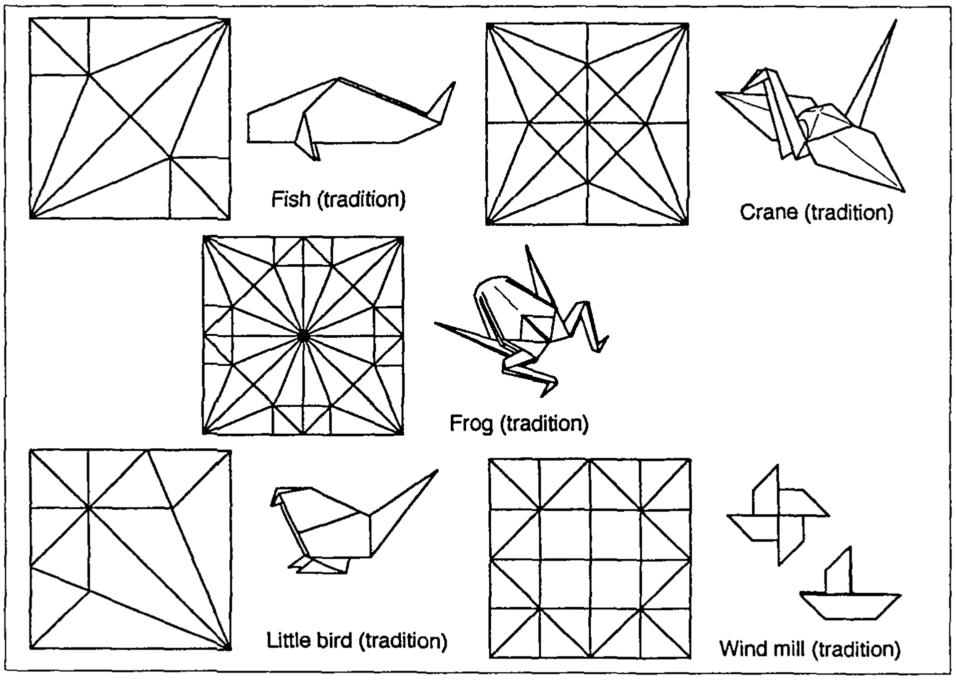


Figure 1

Figure 1 shows some traditional models and their creases. Their primary character reveals a sense of 'easiness'. An easy model stands the strongest chance of survival. However, it is very difficult to define the meaning of 'easiness'. It has, at least, two meanings:

1. Easy to make.
2. Easy to learn.

The former concept relates to technique; the latter to process.

Another keyword in traditional origami is 'natural'. Bearing these two key words in mind, we can now explore the rules of origami in more detail.

Most of the rules (one sheet, no cutting, no gluing, and flat folding) encapsulated in the word 'fold'. These are related to the physical qualities of the paper. We transform a sheet of paper by rolling up, crumpling, folding along curve and folding along straight line. Why flat folding is important in the rule of origami? Miura's studies will give us a hint to solve this problem. He has shown us the peculiar flat folding as a solution of the strength of materials (Miura, 1989). We can find bits of this peculiar flat folding in the traditional models.

As for the characters of traditional origami, we shouldn't ignore the distinction between the surface of paper and the reverse side of it, though it isn't included in Husimi's rules. On most of traditional models, surface and reverse side become outer inner sides as a result that edges of paper are fitted to another edges. It is natural and easy process of origami. In biological terms, it correspond to blastula. It is a wonderful coincidence for me. However, 'origami sheet biology' is very simple. At present, origami creatures are belonging to a kind of *Coelentelata* like a jelly fish.

I may have overlooked the rules of origami. I think the rules of origami and its systematic structures are originate from both physical qualities of paper and 'easiness' as selection pressure. These systematic structures lead me to the taxonomy, and my 'tiling method' is based on the taxonomy.

2 THE ORGANIZATION OF TAXONOMY

There have been some attempts at producing a taxonomy for origami. There are four viewpoints as follows.

1. Process.
2. Symmetry of the complete model.
3. Technique.
4. Structure.

The most famous study on origami taxonomy is that known as 'origami tree'. The pioneer of this study was probably Ohashi (1977). The origami tree was a systematic method used in learning how to construct origami models. Its main concept is the notion of 'basic form'. Basic forms are simple and geometrical forms which can be applied to many different kinds of origami designs. In fact, the crease patterns in Figure 1 aren't actual complete figures, but basic forms of them. There are about 10 basic forms. Returning to the biological analogy, the origami tree is a kind of a genealogical tree, for example, the frog's legs and the Iris's petals are homologous. It is an interesting viewpoint, but it has a rigid aspect because of its adherence to the folding processes.

The symmetrical analysis of complete models is the second type of taxonomy. This view is mentioned by Kihara in his book (Kihara, 1979). He classifies traditional models by point group (a term of crystallography). A new type of work in this field is that by Kawasaki who wasn't aware of Kihara's book. Kawasaki's work is called 'isoarea folding' which is a design of 4-times rotatory inversional symmetry (Kasahara and Takahama).

The third viewpoint considers the origami design to be an assembly of techniques. This view classifies folding techniques. For example, *tsumami ori* (pinch fold), *nejiri* *ori* (twist fold), *sizume ori* (push fold) ... At present, this study is only an idea, but there are many designs which can be explained by their peculiar techniques.

Taxonomy by structure will be described in Sections 3 to 5.

3 ORDERLINESS OF TRADITIONAL MODELS

In Figure 1, there is a systematic pattern in the fish-crane-frog lines. On the other hand, the pattern in the windmill is different from the others. The minimum angle of it is 45 degrees. This pattern and its extension have been given the fitting name of "box pleating" (Lang, 1988; Lang and Weiss, 1990). It has great potential in making new structures. Figure 2 is such an example. However, this type of folding is not always structural because relations between each crease are weak. In short, they are agglutinative.

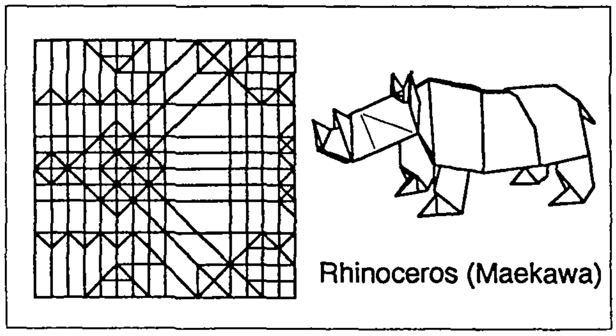


Figure 2

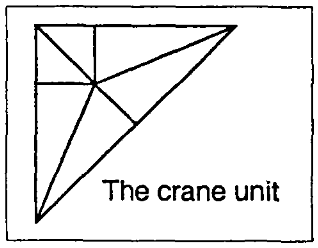


Figure 3

A fundamental shape of the fish-crane-frog system is the right-angle isoscale triangle which is half of the fish base (Fig. 3). I call it the crane unit. The crane base is assembled by 4 units, and the frog base by 8 units. The 8-unit form is not the limit of this system; the other form can be arranged by the same number of units. Figure 4 shows these examples. The flower dish is assembled by the same number of units as the frog units, whereas the bug is assembled by 16 units.

This system has been known for a long time. Uchiyama shows the spider base (4 frog bases = 32 crane units) in his book (Uchiyama, 1979), and it can be traced back more. Figure 5 shows origami designs with cutting from the Edo period (1600-1860's). The upper figure was introduced in *Kayaragusa* (Adachi, 1845). In a slightly different sense, the lower figure is an example of *renkaku* (chained cranes) in *Senbazuru Orikata* (Rokoan, 1797). I have designed other forms using the crane unit assembly. Figure 6 shows an example.

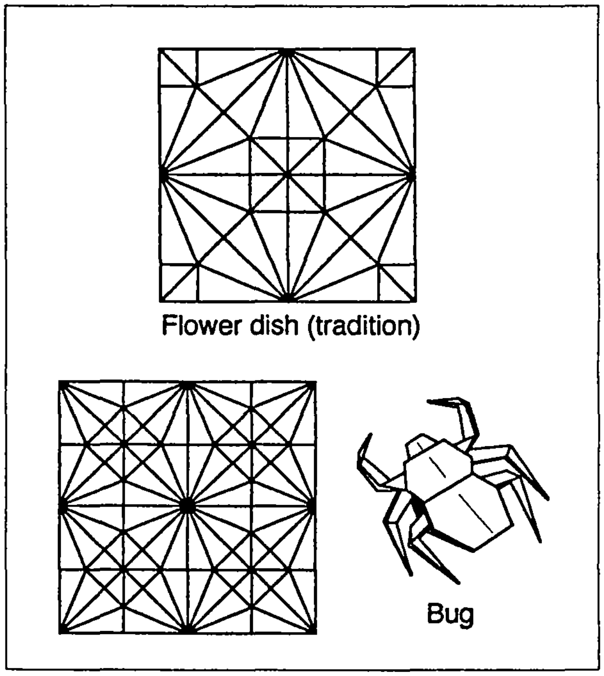


Figure 4

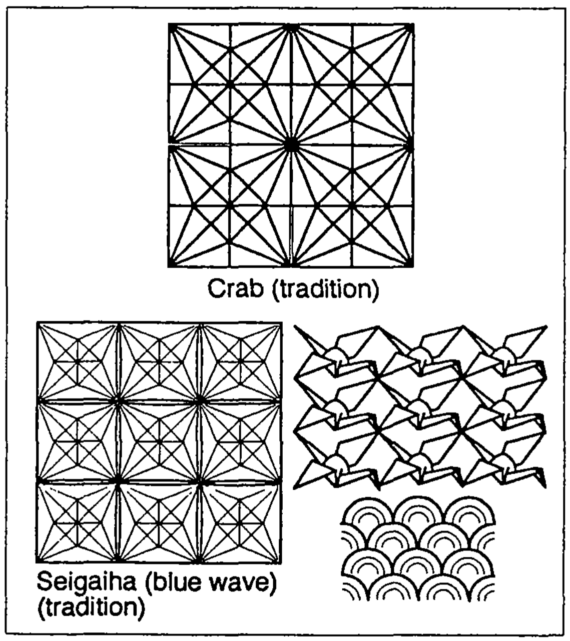


Figure 5

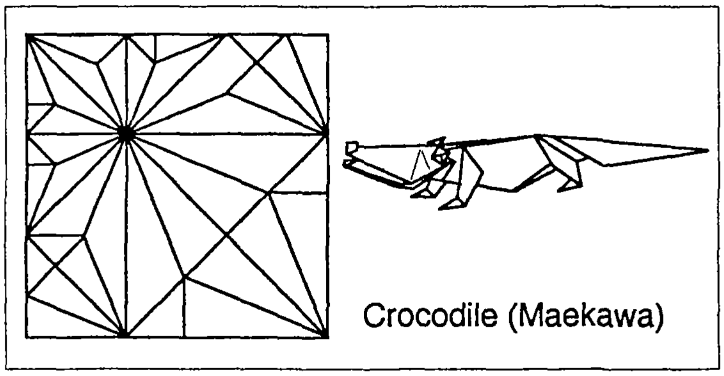


Figure 6

The crane unit is a right-angle isoscale triangle. We can extend this unit to arbitrary triangles. Husimi first explained its geometrical meaning in his book (Husimi and Husimi, 1984).

Husimi's *inner point theorem* states: Any triangle is folded into a form in which all sides are gathered in a straight line. This theorem has been extended to include any quadrilaterals with inscribed circles, and has been extended to quite arbitrary quadrilaterals (Fig. 7).

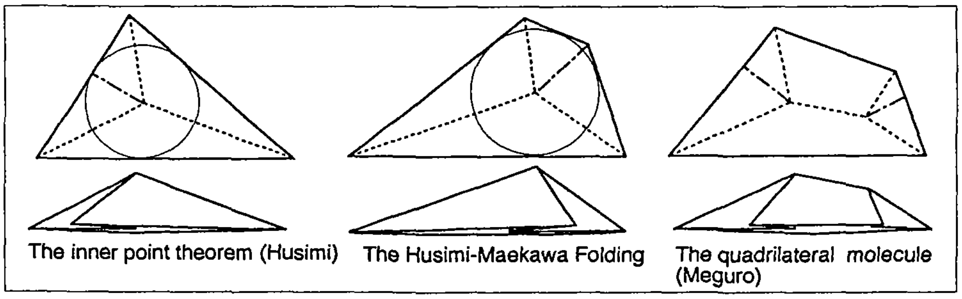


Figure 7

We can make crooked cranes using Husimi's theorem. Since I know the arbitrary triangle unit, I have a tendency to use the right-angle isoscale triangle unit. Using the peculiar triangle, we can design new models easily.

4 ORIENTATION OF THE ELEMENTARY UNITS

The 'crane unit' is not an elementary unit (like an atom) of the crane. It is subdivided here into two types of triangles (Fig. 8). They are elementary units of crane type origami; the little bird base is also assembled by those units.

We can regard the basic forms as the results of conditional tiling work using the elementary units. The following two theorems correspond to conditions of the tiling work, though they aren't sufficient conditions (Kawasaki, 1989).

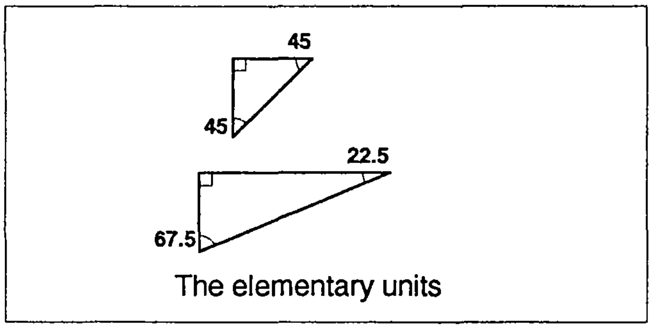


Figure 8

The *Maekawa theorem* states: At any node of a flat folding, except of those on the edge of the plane, the difference of the number of mountain creases and the number of valley creases is equal to two (Maekawa, 1983).

The *Kawasaki theorem* states: At any node of a flat folding, except of those on the edge of the plane, the alternate total of angles between the creases is equal to 180 degrees (Kasahara, 1989; Kawasaki, 1989).

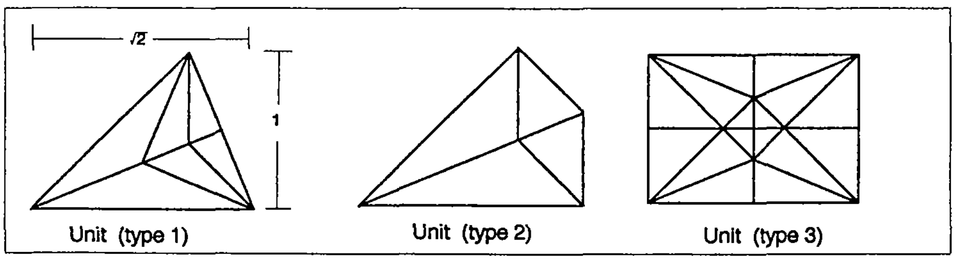


Figure 9

In designing the new model, I do not use the elementary units themselves, but secondary units which are assembled by several elementary units. In short, this tiling work is hierarchical. Figure 9 shows examples of these secondary units. Of course, the crane unit belongs to the group of the secondary units. Recently, a well arranged work using those units was made by Meguro (1991-92). He emphasizes the 'univalency' of the secondary units. In origami, 'univalency' means the character shown in the *Husimi's inner point theorem*, that is to say, that all sides of the figure are gathered in a straight line by flat folding. 'Univalency' is a concept to increase efficiency of the tiling work.

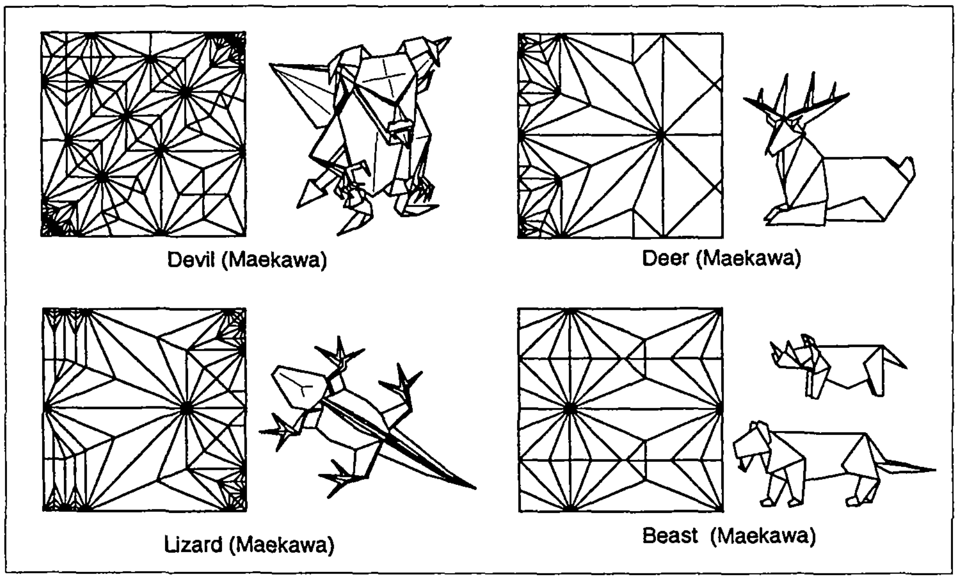


Figure 10

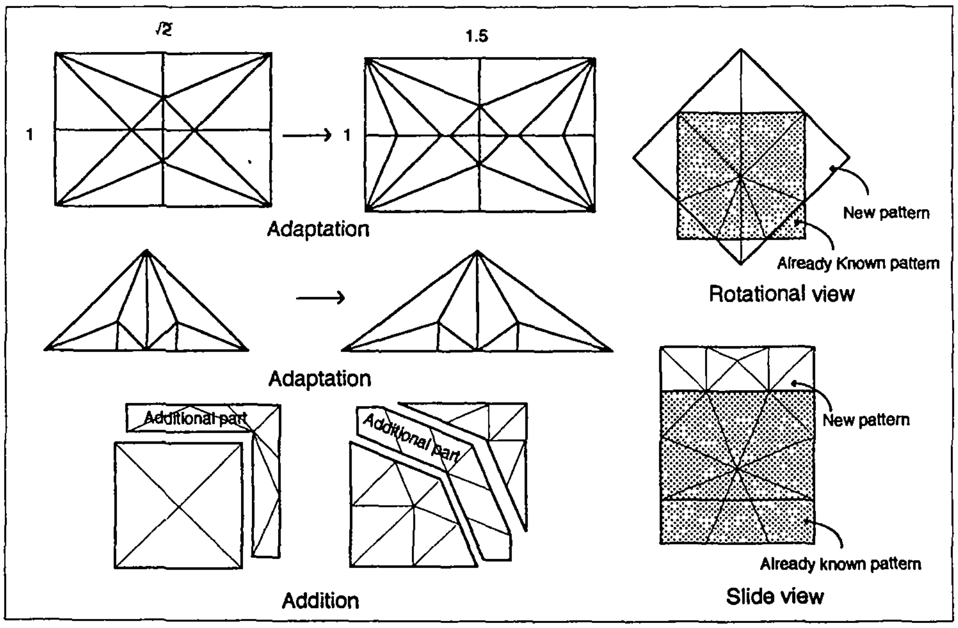


Figure 11

5 THE ORIGINAL DESIGNS

I have created new designs using the tiling work. Figure 10 shows some of these designs. In designing them, I have used various methods – among them: 'adaptation', 'addition', 'rotational view' and 'slide view'. These are illustrated in Figure 11. 'Adaptation' is a re-introduction of arbitrary triangle folding. 'Addition' and others are extended methods of already known forms.

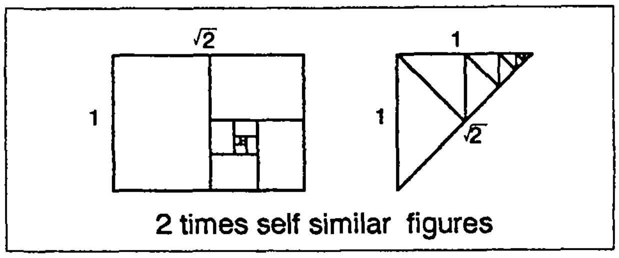


Figure 12

It is interesting that two peculiar figures appear in those forms. One is a rectangle which is square root 2 wide per other side, and the other is a right-angle isoscale triangle. They are figures that can be divided into two self-similar figures as in Figure 12. The crane system is a good example of this pattern. I have achieved some interesting results by starting with a sheet of this peculiar-ratio rectangle paper instead of a sheet of square paper (Fig. 13). (This ratio is very common.) Square root 2 is the magic number of origami, because we find this ratio everywhere.

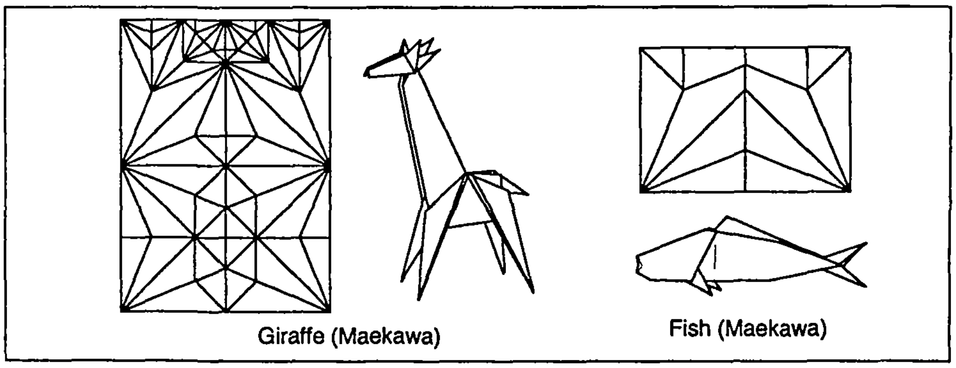


Figure 13

This magic number is significant under the conditions that folding angles are restricted within multiples of 22.5 degrees. If we use other angles, we will find other magic numbers or will not see it.

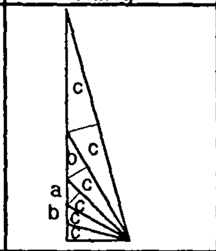
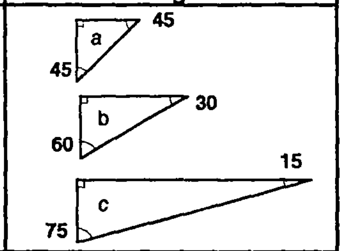
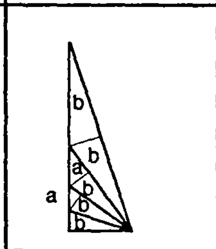
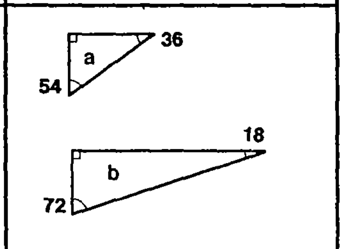
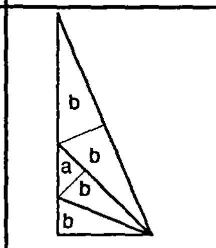
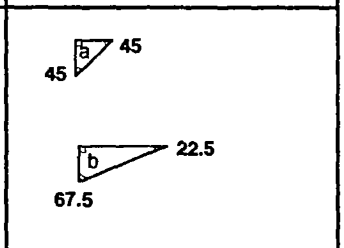
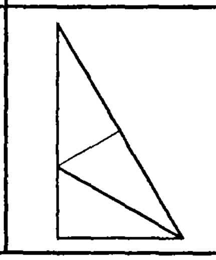
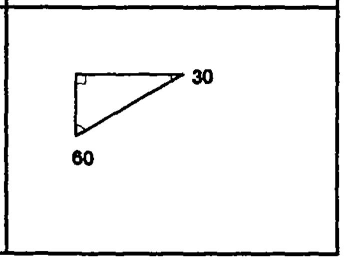
Unit angle	Tiling	Triangles
(1/6)x90degrees		
(1/5)x90degrees		
(1/4)x90degrees		
(1/3)x90degrees		

Table 1

Table 1 shows an extension of the elementary units. The hexasection of the right angle is productive. I have tiled the hexasectional units on a square field as in Figure 14. The trisection has a possibility on regular hexagons and rectangles: the ratio between the width and the length is an integer division or a multiple of the square root of 3. The pentasection can be used on regular pentagons and the Penrose tiles, but I have not accomplished presentable designs to date.

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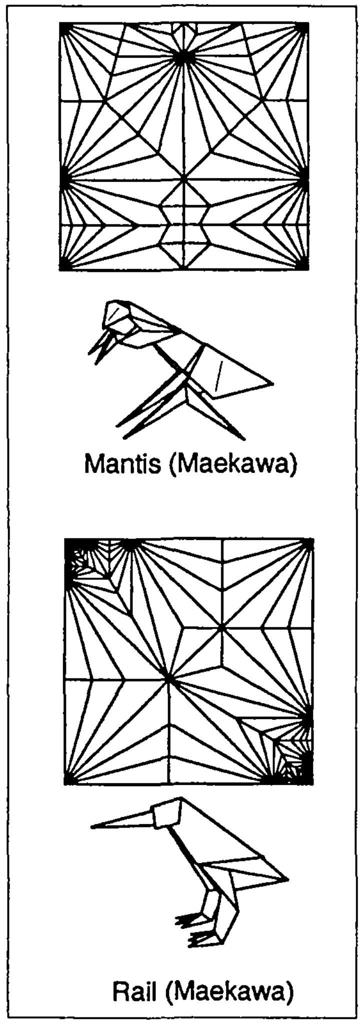

Figure 14